NATIONAL UNIVERSITY OF SINGAPORE

PC4240 – SOLID STATE PHYSICS-II

(Semester II: AY2010-11)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **THREE** questions and comprises **THREE** printed pages.
2. Answer **ALL questions**.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.

1. A metal is placed in a uniform magnetic field H which is applied along the z-axis. Let an AC electric field $E_0e^{-i\omega t}$ be applied perpendicular to H.

   (a) If the electric field is circularly polarized ($E_y = \pm iE_x$), show that the current density along the x axis can be written as

   $$J_x = \left(\frac{\sigma_0}{1 - i(\omega \mp \omega c)\tau}\right) E_x$$

   $$J_y = \pm iJ_x$$

   $$J_z = 0$$

   (b) Show solutions to Maxwell equations are

   $$E_x = E_0 e^{i(kz-\omega t)},\ E_y = \pm iE_x,\ E_z = 0$$

   provided $k^2c^2 = \varepsilon\omega^2$ and the frequency dependence of the dielectric constant ε is given by

   $$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega}\left(\frac{1}{\omega \pm \omega_c + i/\tau}\right)$$

   where ω_p is the plasma frequency and ω_c is the cyclotron resonance frequency.

   (c) Show that when $\omega \ll \omega_c$, the relation between k and ω for the low-frequency solution is

   $$\omega = \omega_c\left(\frac{k^2c^2}{\omega_p^2}\right)$$

   This low frequency wave, known as a helicon, has been observed in many metals.

2. Consider a sample consisting of N electrons each with spin s = ½ and magnetic moment μ_B. The sample is subjected to a static magnetic field B. The spins interact with the applied field but not among themselves.

   (a) Find the average magnetic moment and energy of the sample. Assume that the spins are in thermal equilibrium at temperature T.

   (b) Find the entropy and heat capacity of this sample.

(c) The sample in thermal equilibrium with a reservoir at T = 1 K, in a magnetic field of B = 1 T. The sample is then thermally isolated from the reservoir and the field is reduced to B = 0.01 T. What will be the final temperature reached?

(d) Repeat parts (a)-(c) for spin s = 1

3. (a) Derive the London equation for a superconductor and explain how it leads to the Meissner effect.

(b) Find an expression for magnetization M(x) under an external magnetic field $\vec{B} = B_0\vec{e}_z$ for a superconducting slab of thickness $a \ll \lambda_L$, where λ_L is the London penetration depth.

(c) Show that a thermally isolated superconducting sample at a temperature $T = \alpha T_C$ (α<1) will cool if an external magnetic field $B > B_C(T)$ is applied. T_C and B_C are critical temperature and critical field, respectively. Find the corresponding drop in the temperature.

(d) The superconductor tin (Sn) has T_C = 3.7 K and critical field of B_C = 30.6 mT at T = 0 K. Calculate the critical current for a tin wire of diameter 1mm at T = 2 K. What diameter of a wire would be required to carry a current of 100 A?

---End of the paper---

(R.M)